Name: Section: Student No:

Show your work! Give the best result that you can give!
Each question worth 15 points unless stated otherwise

NOTICE TO THE STUDENTS

Read the instructions carefully listed below and sign the box:
1.Textbooks, lecture notes, calculators with extensive memories, and any kind of computers are not permitted in the classrooms: if you have any, leave them on the instructors desk.
2.Cell phones should be totally switched off (not in silent or flight modes) and do not keep them with you: either put them in your bags or leave them on the instructors desk.
3. Permitted material to be kept on your desks are; pencils, sharpeners, erasers (and in case you may need: water and tissues). Pencil boxes are strictly forbidden.
4. Check your desk for any graffiti; the graffiti related to the course will be treated as an attempt to cheat.
5. You are not allowed to talk to other students during the exam whatever the reason may be.
6. Disobeying the above rules will be severely penalized and a disciplinary action will be conducted.
7. Please prepare your IDs (with photos) on your desk for identity check.

(to be signed when exam is finished!)
I certify that this is my own work only.

Name: ....
Signature: ....

Time: ....

1. Given General Knapsack Problem:

$$\textbf{MP(b)} \quad \begin{array}{lll} \max & 7x_1 + x_2 + 4x_3 + 10x_4 + 17x_5 & \\ \text{such that} & 3x_1 + x_2 + 2x_3 + 4x_4 + 6x_5 & = b \\ & x_j \in \{0, 1, 2, 3, \ldots\} \quad \forall j & \end{array}$$

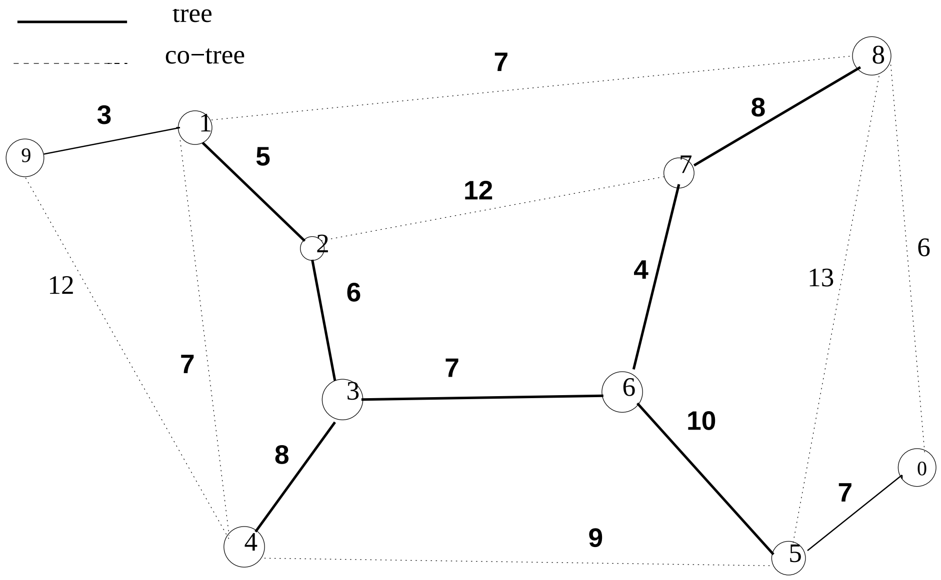
A solution is given with the Table:

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$F(t)$	0	1	4	7	10	11	17	18	21	24	27	28	34	35	38	41	44	45	51	52	55
$p(t)$	–	2	3	1	4	1	5	1	3	1	4	1	5	2	3	5	4	1	5	2	3

Find/Draw the longest path tree!

And write solution for t=17, and t=15 as $X = (x_1, x_2, x_3, x_4, x_5)$.

2. Given the graph with dark edges as tree and dotted edges as co-tree edges:

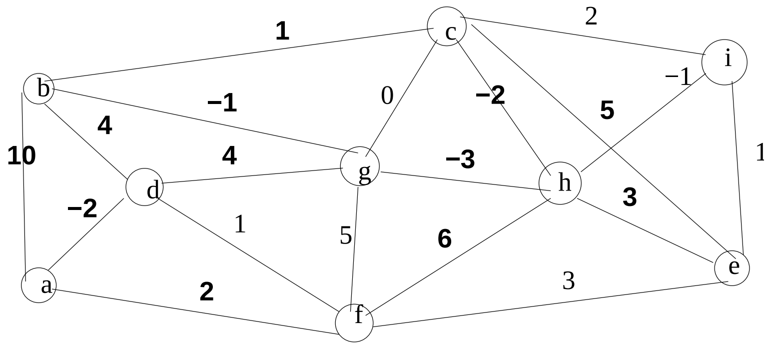


- Determine fundamental Cycle C(T,e) for e=(0,8) (i.e. list the edges) **2 pts**

- Determine fundamental Cocyle D(T,f) for f=(2,3) (i.e. list the edges) **2pts**

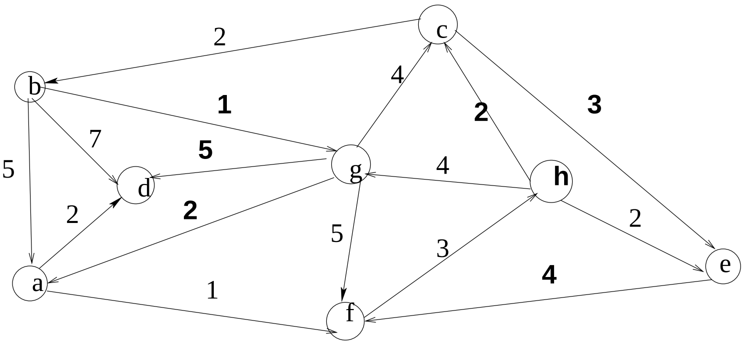
- By using fundamental cocycles, check for optimality of T; and find an optimal tree by switch a tree edge with a co tree edge until finding an optimal tree. Show your calculations **9 pts**

3. Given the graph G with root r, apply Prim-Dijkstra algorithm for the Minimum Spanning Tree Problem (MST). You can implement as you desire. Fill the following table. Where iter means iteration, 'node' means node added to tree, 'edge' means edge=(i,j) with i in X and j in X^c, 'weight' means w(i,j) cost of the edge selected, and 'total' means total cost of the selected edges. Draw the final tree! **Let r=h**



iter.	node	edge	weight	total
0	r	-	-	0
1				
2				
3				
4				
5				
6				
7				
8				
9				

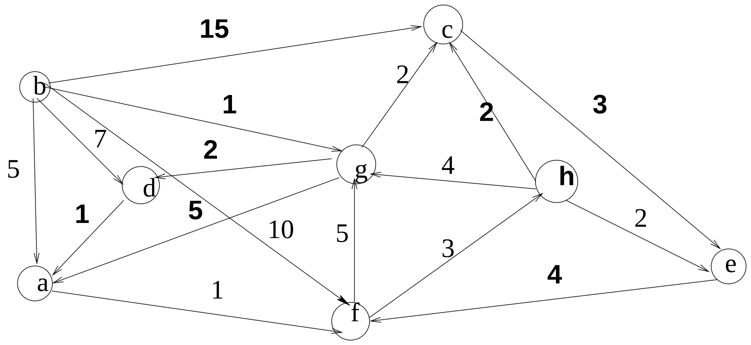
4. Consider the Shortest Path Problem for the directed graph G given below, with selected r=b. Apply 4 iterations of Dijkstra's Shortest Path Algorithm. Fill the following table,



iter	node	edge	parent	dist	total	a	b	c	d	e	f	g	h
						∞	0	∞	∞	∞	∞	∞	∞
0	$r = b$	$-$	$-$	0	0								
1													
2													
3													
4													
5													

Where 'iter' mean iteration count, 'node' i is the node added and to be scanned, 'edge' is the edge added to Tree, 'parent' is p(i); parent of the node which is added to tree, 'dist' is the distance from r to i (shortest distance), and 'total' is the sum of distances of nodes in the current tree T. The right part of table should contain d(j)/p(j) values. When they are fixed they will move to left part.

5. Consider the Shortest Path Problem for the directed graph G given below, with selected r=b. Apply 4 iterations of Bellman-Ford Algorithm. Fill the following table,



iter k	1/a	2/b	3/c	4/d	5/e	6/f	7/g	8/h
$k = 0$	∞	0	∞	∞	∞	∞	∞	∞
$k = 1$								
$k = 2$								
$k = 3$								
$k = 4$								

6. **Union-Find** Starting with forest of $\{1, 2, \ldots, 15\}$ each with rank=0, apply following union operations a=(5,6), b=(7,8), c=(1,2), d=(3,4), e=(6,8), f=(2,3), g=(9,11), h=(10,12), i=(11,10), j=(10,13), k=(9,2), l=(5,1),m=(8,13) n=(15,5). When taking union of two equal rank sets choose the one whose root has smallest index as the new root. Use also path compression at each iteration. Mark any edge which is cut from the tree with x. **10**

Also write down final rank of the nodes

node	1	3	5	7	9	11	13
rank							

7. **0-1 Knapsack Problem**

Consider the 0-1 Knapsack Problem

$$\textbf{F(t)} \quad \begin{array}{lll} \max & \sum_{j=1}^{j=n} w_j x_j & \\ \text{such that} & \sum_{j=1}^{j=n} a_j x_j & \leq \quad t \\ & x_j \in \{0,1\}, \ \text{for each } j & \end{array}$$

Define $F_k(t)$ as

$$F_k(t) = \max\{\sum_{j=1}^{j=k} w_j x_j : \sum_{j=1}^{j=k} a_j x_j \leq t, x_j \in \{0,1\}\}$$

Assume that $F_k(0) = 0$ for all k, $F_o(t) = 0,$ for $t \geq 0$, and $F_k(t) = -\infty$ for $t < 0, k \geq 0$.

i) Write down a recurrence relation for $F_k(t)$ involving $F_k(.), a_k, w_k, t$ **1 pt**

$$F_k(t) = \max\{F_{k-1}(t), F_{k-1}(t - a_k) + w_k$$

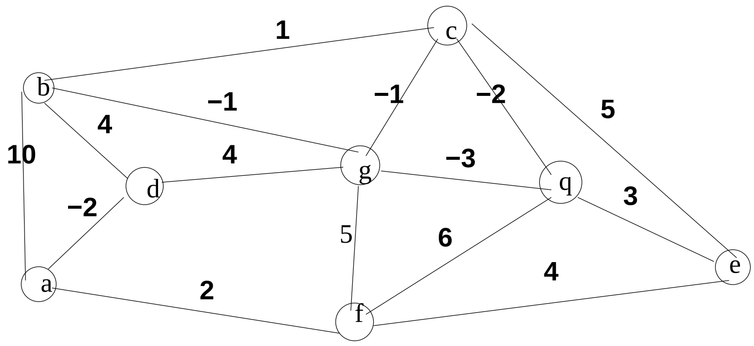
i) Given the following 0-1 Knapsack Problem solve it by applying a recurrence relation and filling the following table. You need to store values of $F_k(t)$, for missing values of k and t. Also you need to store value of x_k in evaluating $F_k(t)$. Show it as $F_k(t)/x_k$ Such as 5 / 0 or 5 / 1. **7 pts**

k=5, w = (3, 4, 6, 10, 20), a = (2, 3, 4, 4, 5), t= 10

k/t	0	1	2	3	4	5	6	7	8	9	10
k=0	0	0	0	0	0	0	0	0	0	0	0
k=1	0/0	0/0	3/1	3/1	3/1	3/1	3/1	3/1	3/1	3/1	3/1
k=2	0/0	0/0	3/0	4/1	4/1	7/1	7/1	7/1	7/1	7/1	7/1
k=3	0/0	0/0	3/0	4/0	6/1	7/0	9/1	10/1	10/1	13/1	13/1
k=4	0/0	0/0	3/0	4/0	10/1	10/1	13/1	14/1	16/1	17/1	
k=5	0/0	0/0	3/0	4/0	10/0						

ii) determine the solution vector $x = (x_1, x_2, x_3, x_4, x_5)$ for t=9 and t=10 **4 pts**

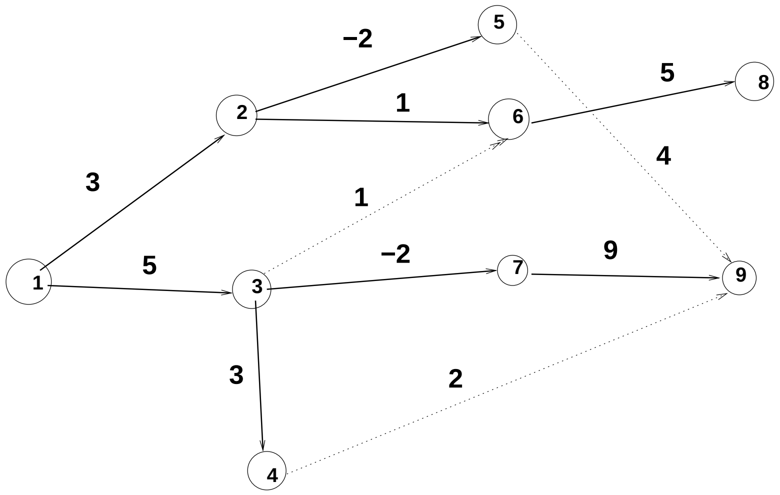
8. Given the undirected graph with edge weights, for the minimum spanning tree problem



Apply 7 iterations of Greedy/Kruskal's Algorithm. Indicate 7 edges that is processed, and indicate with + or - whether that edge will be part of optimal tree ? Draw the resulting forest ! **10 pts**

index	1	2	3	4	5	6	7
edge (i,j)							
weight							
selected +/-							

9. Given G as T and $T^{\perp}$ for the shortest path problem with root r=1, is T optimal? Verify it, or find an optimal tree by improving with Network Simplex Method, if there is one. Compute shortest distances and parent pointers in the final tree. **10**



k	1	2	3	4	5	6	7	8	9
d	0								
p	-								