

You will be assigned one project from below. You will write a program to the given problem/algorithm. You will test with several input data. You will use C and compile and run on Liste.ctis.bilkent.edu.tr. We will test your program after compiling on liste machine, run test with supplied data and new data. Deadline will be Jan 9 2014 .

### 1. Prim -Dijkstra Algorithm for MST

Given an undirected graph  $G=(V,E)$   $V=\{1,2, .. n\}$ ,  $E=\{1,2, ..., m\}$ . Graph will be given as  $n$ ,  $m$  and list of  $m$  edges. Each edge will be given endpoints  $i$  and  $j$  for edge  $e=(i,j)$  and weight  $w_e = w_{i,j}$ .

User will input root node:  $r$

Then algorithm will find a minimum spanning tree for root  $r$ , say  $T$ .  $T$  will be listed two ways: i) a list of edges, ii) a table showing parent pointers, with  $p(r)=-1$ . It will also output cost of  $T$ .

You need to represent tree with parent points only. Compute and lists depth of the nodes in  $T$ .

### 2. Fundamental Cycles

Given an undirected graph  $G=(V,E)$   $V=\{1,2, .. n\}$ ,  $E=\{1,2, ..., m\}$ . List all fundamental cycles with respect to  $T$ . Graph will be given as  $n$ ,  $m$  and list of  $m$  edges. Each edge will be given endpoints  $i$  and  $j$  for edge  $e=(i,j)$  and  $i < j$  .

User will input root node:  $r$ . Edge list will be given in two parts:  $T$  and  $T^\perp$ . You need to determine parent pointers of  $T$  with root  $r$ .

The algorithm will output  $T$  as directed tree with root  $r$  via parent pointers  $p(i)$  and depth/height numbers  $h(i)$ . You may use breadth first search among edges of  $T$ .

Then algorithm will output for each edge  $e \in T^\perp$ ,  $e$  and edges of  $C(T,e)$ .

Also number edges of  $T$  as  $1, 2, n-1$ , and edges of  $T^\perp$  as  $n, .., m$ . You need to form a matrix where rows are numbered  $1$  to  $n-1$  and columns are  $n, .., m$  and  $C(i, j) = 1$  if  $i \in C(T, j)$ ,  $= 0$  otherwise .

The program should output matrix  $C$  also.

### 3. MST Tree with Greedy (Kruskal's algorithm).

Given an undirected graph  $G=(V,E)$   $V=\{1,2, .. n\}$  ,  $E=\{1,2, ..., m\}$ . Graph will be given as  $n$ ,  $m$  and list of  $m$  edges. Each edge will be given end endpoints  $i$  and  $j$  for edge  $e=(i,j)$  and weight  $w_e = w_{i,j}$ .

We want to find a spanning tree whose total weight is maximum. You need to use heap to find edges with maximum weight and use union-find algorithm to check whether  $F+e$  contains a cycle. You start with Forest  $F = (V, \emptyset)$  and stop with tree  $T$  containing all nodes or of size  $|V| - 1$ .

#### 4. Shortest Path with Dijkstra Algorithm

Given a directed graph with non-negative weights, and given  $r$ , you need to find a shortest path tree with  $d(i)$ ,  $p(i)$  for each node, where  $d(i)$  length of a shortest path from  $r$  to  $i$ , and  $p(i)$  parent of node  $i$ . Parent of root is set to  $-1$ . Nodes are numbered with positive integers.

Input consists of:

$n$  number of nodes, for each node  $i$ , edges originating from  $i$  is given as

$i \ j \ w_{i,j} \ k \ w_{i,k} \ \dots \ -1$

a node, say  $k$ , without any edges leaving it represented as

$k \ -1$

After inputting the graph, The program will prompt for root  $r$  and find the shortest path tree and at each iteration will print each node with  $d(i)$  and  $p(i)$ . At the end will print the the triples  $(i, d(i), p(i)) \ i=1, \dots, n$ .

#### 5. Bellman-Ford Algorithm

Given a directed graph with arbitrary integer weights, and given  $r$ , you need to find a shortest path tree with  $d(i)$ ,  $p(i)$  for each node, where  $d(i)$  length of a shortest path from  $r$  to  $i$ , and  $p(i)$  parent of node  $i$ . Parent of root is set to  $-1$ . Nodes are numbered with positive integers.

Input consists of:

$n$  number of nodes, for each node  $i$ , edges originating from  $i$  is given as

$i \ j \ w_{i,j} \ k \ w_{i,k} \ \dots \ -1$

a node, say  $k$ , without any edges leaving it represented as

$k \ -1$

After inputting the graph, The program will prompt for root  $r$  and You will start with  $d(r)=0$  and  $d(i) = \infty, i \neq r$ , and  $p(r) = -1, p(i) = 0$ . You will apply Bellman-Ford Algorithm at most  $n$  iterations. You can stop earlier if  $d(i)$ 's become stable or  $d(r)$  becomes negative.

The program should out triple  $(i, d(i), p(i)) \ i=1, 2, \dots, n$ .

## 6. Network Simplex Method for Shortest path

Given a directed graph with arbitrary integer weights, and given  $r$ , you need to find a shortest path tree with  $d(i)$ ,  $p(i)$  for each node, where  $d(i)$  length of a shortest path from  $r$  to  $i$ , and  $p(i)$  parent of node  $i$ . Nodes are numbered with positive integers.

Input consists of:

$n$  number of nodes, for each node  $i$ , edges originating from  $i$  is given as

$i \ j \ w_{i,j} \ k \ w_{i,k} \ \dots \ -1$

a node, say  $k$ , without any edges leaving it represented as

$k \ -1$

Let us add node numbered 0 as a dummy node, let  $r$  be real root. Let  $T_o$  be defined via  $d(r)=0$ , and  $d(i) = 1000, i > 0, i \neq r$ ,  $d(0)=0$ ,  $p(0)=-1$ ,  $p(i) = 0, i > 0$ . Apply network simplex method until a) you find an an optimal solution to Shortest path problem rooted at node  $o$  or  $r$ , b) find a negative directed cycles.

The Program should output, in a) triples  $(i, d(i), p(i))$  in b) in addition give additional info for negative cycle.

## 7. Fundamental Co-Cycles

Given an undirected graph  $G=(V,E)$   $V=\{1,2, \dots, n\}$ ,  $E=\{1,2,\dots, m\}$ . List all fundamental cocycles with respect to  $T$ . Graph will be given as  $n$ ,  $m$  and list of  $m$  edges. Each edge will be given endpoints  $i$  and  $j$  for edge  $e=(i,j)$  and  $i < j$ .

Edge list will be given in two parts:  $T$  and  $T^\perp$ . root and parent pointers of  $T$  will be given as an array. parent of node will be given as  $-1$ . all node numbers and edge numbers will be given as positive integers.

Then algorithm will output for each edge  $f \in T$ ,  $f$  and edges of  $D(T,e)$ .

Also number edges of  $T$  as  $1, 2, \dots, n-1$ , and edges of  $T^\perp$  as  $n, \dots, m$ . You need to form a matrix where rows are numbered  $1, 2, \dots, n-1$  and columns are  $n, \dots, m$  and  $B(i, j) = 1$  if  $i \in D(T, j)$ ,  $= 0$  otherwise.

The program should output matrix  $B$  also.

## 8. Topological Graphs

Given Directed Graph  $G=(V,E)$  determine whether  $G$  is a directed graph. As output either you will give topological numbering, or output a directed cycle.

You can choose any data structure for representing  $G$ .

## 9. General Knapsack Problem

Given vectors,  $w$ ,  $a$  and total resource  $b$ , give solution of problem

$$\max\{wx : ax \leq b; x_j \in N\}$$

You need to determine value of the objection function, solution vector  $x(b)$  and algorithm which gives  $x(t)$  for  $t \leq b$ .