

Name:

Section:

Student No:

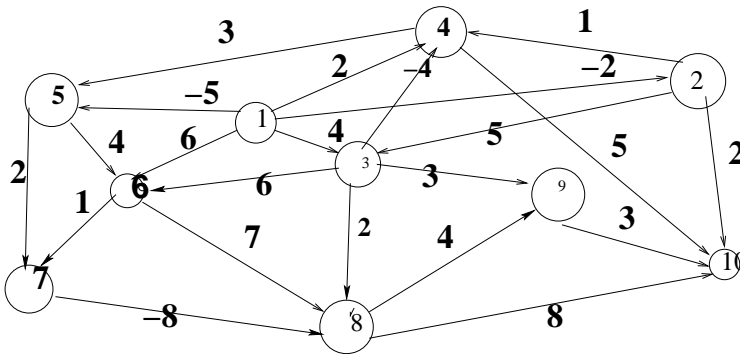
Closed Book, closed note exam. Show your work! we must follow your reasoning. Give the best result that you can give! Over 100 points is bonus.

SIGNATURE

Time of Submission:

1. Using the given topological numbering and using numbers on the edges as edge costs $w(i,j)$ solve **the longest path problem** with root 1. Show the solution tree **2 pt**

Note that Longest path problem is to find for each node $k > 1$, find the longest path from node 1 to node k , using directed edges and not visiting any nodes more than once.



Fill the following table also :

k	1	2	3	4	5	6	7	8	9	10
d(k)	0									
p(k)	-									

2. Given the 0-1 Knapsack problem with following parameters and table

$k=5$, $w = (3, 4, 6, 10, 20)$, $a = (2, 3, 4, 4, 5)$, $t=9$

k/t	0	1	2	3	4	5	6	7	8	9
k=0	0	0	0	0	0	0	0	0	0	0
k=1	0/0	0/0	3/1	3/1	3/1	3/1	3/1	3/1	3/1	3/1
k=2	0/0	0/0	3/0	4/1	4/1	7/1	7/1	7/1	7/1	7/1
k=3	0/0	0/0	3/0	4/0	6/1	7/0	9/1	10/1	10/1	13/1
k=4	0/0	0/0	3/0	4/0	10/1	10/1	13/1	14/1	16/1	17/1
k=5	0/0	0/0	3/0	4/0	10/0	20/1	20/1	23/1	24/1	30/1

Determine the solution vector $x = (x_1, x_2, x_3, x_4, x_5)$ for $t=9$ **2 pts**

3. Given General Knapsack Problem:

$$\begin{array}{ll}
 \text{max} & 7x_1 + x_2 + 4x_3 + 10x_4 + 17x_5 \\
 \text{MP(b)} \quad \text{such that} & 3x_1 + x_2 + 2x_3 + 4x_4 + 6x_5 = b \\
 & x_j \in \{0, 1, 2, 3, \dots\} \quad \forall j
 \end{array}$$

A solution is given with the Table:

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
$F(t)$	0	1	4	7	10	11	17	18	21	24	27	28	34	35	38	41	44	45	51	52	55		
$p(t)$	–	2	3	1	4	1	5	1	3	1	4	1	5	2	3	5	4	1	5	2	3		

- Write a recurrence relation for $F(t)$ **explicitly; i.e. involving t and F only.** 1 pts
- Find the solution vector $X = (x_1, x_2, x_3, x_4, x_5)$, $X(15)$ for $MP(15)$, 2 pts

4. **LCS - largest Common Subsequence**. Given the sequence $X = (B, A, C, D, A, B, A)$ and $Y = (A, B, C, A, D, B, A, A)$, we want to find largest subsequence common to both sequences X and Y. The tableau of $c(i,j)$ is given as

i	j	0	1	2	3	4	5	6	7	8
	y_j		A	B	C	A	D	B	A	A
0	x_i	0	0	0	0	0	0	0	0	0
1	B	0	0 \uparrow	1 \swarrow	1 \leftarrow	1 \leftarrow	1 \leftarrow	1 \swarrow	1 \leftarrow	
2	A	0	1 \swarrow	1 \uparrow	1 \uparrow	2 \swarrow	2 \leftarrow	2 \leftarrow	2 \swarrow	
3	C	0	1 \uparrow	1 \uparrow	2 \swarrow	2 \uparrow	2 \uparrow	2 \uparrow	2 \uparrow	
4	D	0	1 \uparrow	1 \uparrow	2 \uparrow	2 \uparrow	3 \swarrow	3 \leftarrow	3 \leftarrow	
5	A	0	1 \swarrow	1 \uparrow	2 \uparrow	3 \swarrow	3 \uparrow	3 \uparrow	4 \swarrow	
6	B	0								
7	A	0								

- i) Fill the empty cells in row 6 (except column 8) **2 pts**
- ii) Find a largest common subsequence of X^5 and Y^7 **2 pts**
 (Note: X^5 is the subsequence of X containing first 5 elements.)