CTIS264 Quiz 1 Solution 26/09/13

Name: Section: Student No:

Closed Book, closed note exam. Show your work! we must follow your reasoning. Give the best result that you can give! Over 100 points is bonus.

SIGNATURE	. . . .	Time of Submission:

1. Suppose you have 4 algorithms $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$, $\mathcal{A}_4$ all for the same problem. Assume, all input can be divided into 5 groups, say, X_1, X_2, X_3, X_4, X_5 and you happen to know performance, in terms of running time, of these algorithms over all possible inputs of size n. for each group.

	X_1	X_2	X_3	X_4	X_5
$\mathcal{A}_1$	$5n^6 \log n + n^{7/2}$	$1000n^4 + n^3 \log n$	$0.01n^2 + 2^n$	$n^6 + 10^{10}n^2 \log n$	$n^{100} \log n + 300n^7$
$\mathcal{A}_2$	$1.05n^3 \log n$	$100n^3 \log n$	$125n^2 \log n + n^3 \log n$	$n \log n + 0.001n^3 \log n$	$n^2(\log n)^2$
$\mathcal{A}_3$	$10^6 n^2$	$0.00001n^2$	$n^2 + 10^{10}n + 5n \log n$	$500n^2 + 100n \log n$	$1000n^2 + n(\log n)^3$
$\mathcal{A}_4$	$100n^{50} + 200n^5$	$n^3 \log n + 50n^2$	e^n	$n^3 + 100n \log n$	$50n^2 \log n$

Simplyfy above expressions by identifying dominating term :

	X_1	X_2	X_3	X_4	X_5
$\mathcal{A}_1$	$n^6 \log n$	n^4	2^n	n^6	$n^{100} \log n$
$\mathcal{A}_2$	$n^3 \log n$	$n^3 \log n$	$n^3 \log n$	$n^3 \log n$	$n^2(\log n)^2$
$\mathcal{A}_3$	n^2	n^2	n^2	n^2	n^2
$\mathcal{A}_4$	n^{50}	$n^3 \log n$	e^n	n^3	$n^2 \log n$

Give performance of $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$, $\mathcal{A}_4$ in terms of $O(.), \Omega(.), \Theta(.)$ notation. **4 pts**

	$O()$	$\Omega()$	$\Theta()$
$\mathcal{A}_1$	2^n	n^4	$--$
$\mathcal{A}_2$	$n^3 \log n$	$n^2(\log n)^2$	$--$
$\mathcal{A}_3$	n^2	n^2	n^2
$\mathcal{A}_4$	e^n	$n^2 \log n$	$--$

2. Assume X is an algebraic object whose multiplication is relatively expensive. We want to compute X^n for various n. **Explicitly state # number of multiplications. Justify**

   Assume you can store as many X^j as you wish, how would you compute X^n for n=97, 263 . Indicate which X^j you store and how do you compute desired X^n's ? (Show which multiplications) **2 pts**

   n=97=64+32+1=$2^6 + 2^5 + 1$. $X^n = X^{2^6} * X^{2^5} * X \Longrightarrow$ 6+2=8 multiplications

   n=268=256+4+2+1=$2^8 + 4 + 2 + 1$ $X^n = X^{2^8} * X^4 * X^2 * X \Longrightarrow$ 8 +3 = 11 multiplications

3. Solve the following recursion via Master Theorem , if possible. Show your calculations and justify your results. **5 pts**

   - $$T(n) = 2T(n/3) + \Theta(n^2)$$
     $\alpha = \log_3 2 < 1 < \beta = 2$. T(n)=$O(n^\beta) = O(N^2)$
   - $$T(n) = 6T(n/2) + \Theta(n^2)$$
     $\alpha = \log_2 6 > 2 = \beta$. T(n)=$O(n^\alpha) = O(n^{\log_2 6})$
   - $$T(n) = T(n/4) + \Theta(\log^2 n)$$
     $\alpha = \log_4 1 = 0 = \beta$, k=2 . T(n)=$O(f(n) \log n) = O(\log^3 n)$
   - $$T(n) = 2T(n/3) + \Theta(n \log n]$$
     $\alpha = \log_3 2 < 1 = \beta$, k=1, Master Theorem does not apply
   - $$T(n) = 3T(n/3) + \Theta(n \log n]$$
     $\alpha = \log_3 3 = 1 = \beta$, k=1. T(n)=$O(f(n) \log n) = O(n(\log n)^2)$