CTIS264 Quiz 3 21/11/13

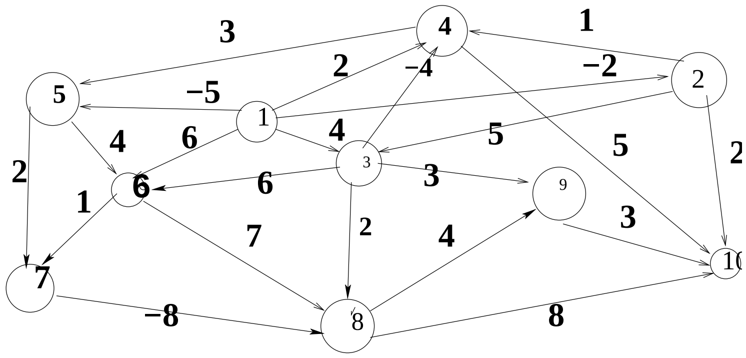
Name: Section: Student No:

Closed Book, closed note exam. Show your work! we must follow your reasoning. Give the best result that you can give! Over 100 points is bonus.

SIGNATURE	. . . .	Time of Submission:

1. Using the given topological numbering and using numbers on the edges as edge costs w(i,j) solve **the longest path problem** with root 1. Show the solution tree **1 pt**

   Note that Longest path problem is to find for each node $k > 1$, find the longest path from node 1 to node k, using directed edges and not visiting any nodes more than once.



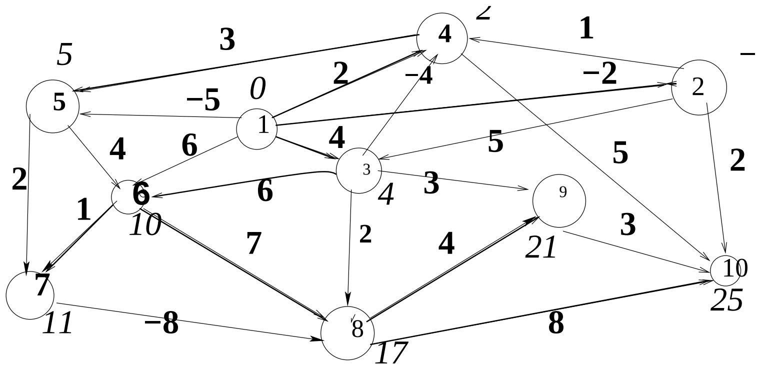
we use the formula:

$$d(1) = 0, for k = 2, ..n; d(k) = \max_i \{d(i) + w_{i,k} : i < k\}, \text{ with defining } i \text{ as } p(k).$$

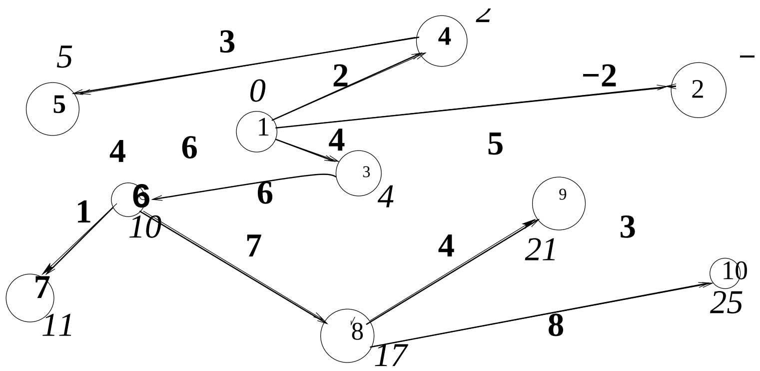
Fill the following table also :

k	1	2	3	4	5	6	7	8	9	10
d(k)	0	-2	4	2	5	10	11	17	21	25
p(k)	-	1	1	1	4	3	6	6	8	8

Tree edges are shown in bold



Just Tree edges and distances



2. Given the 0-1 Knapsack problem with following parameters and table

   k=5, w = (3, 4, 6, 10, 20), a = (2, 3, 4, 4, 5), t= 9

k/t	0	1	2	3	4	5	6	7	8	9
k=0	0	0	0	0	0	0	0	0	0	0
k=1	0/0	0/0	3/1	3/1	3/1	3/1	3/1	3/1	3/1	3/1
k=2	0/0	0/0	3/0	4/1	4/1	7/1	7/1	7/1	7/1	7/1
k=3	0/0	0/0	3/0	4/0	6/1	7/0	9/1	10/1	10/1	13/1
k=4	0/0	0/0	3/0	4/0	10/1	10/1	13/1	14/1	16/1	**17/1**
k=5	0/0	0/0	3/0	4/0	10/0	20/1	20/1	23/1	24/1	30/1

Determine the solution vector $x = (x_1, x_2, x_3, x_4, x_5)$ for t=9 **2 pts**

For $t > 0$ and $k > 0$, Let $F_k(t) = \alpha/\beta \Rightarrow x_k = \beta$ and $t \Longleftarrow t - a_k x_k$.

t=9, k=5, $F_5(9) = 30/1 \Rightarrow x_5 = 1$, and $t \leftarrow 9 - a_5 = 9 - 5 = 4$
t=4, k=4, $F_4(4) = 10/1 \Rightarrow x_4 = 1$, and $t \leftarrow 4 - 1_4 = 0$.
Since t=0, then $x_1 = x_2 = x_3 = 0$. Thus $x = (0, 0, 0, 1, 1)$. Notice that $a_4 + a_5 = 9$ and $w_4 + w_5 = 25$ verifying validity of solution.

3. Given General Knapsack Problem:

$$\textbf{MP(b)} \quad \begin{array}{lll} \max & 7x_1 + x_2 + 4x_3 + 10x_4 + 17x_5 & \\ \text{such that} & 3x_1 + x_2 + 2x_3 + 4x_4 + 6x_5 & = b \\ & x_j \in \{0, 1, 2, 3, \ldots\} \quad \forall j & \end{array}$$

A solution is given with the Table:

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
$F(t)$	0	1	4	7	10	11	17	18	21	24	27	28	34	35	38	41	44	45	51	52	55	**58**	**61**
$p(t)$	$-$	2	3	1	4	1	5	1	3	1	4	1	5	2	3	5	4	1	5	2	3	**1**	**4**

- Write a recurrence relation for F(t) **explicitly; i.e. involving t and F only**. **1 pts**

$$F(t) = \max_{1 \leq k \; 5}\{F(t - a_k) + w_k\} = \max\{F(t-3)+7, F(t-1)+1, F(t-2)+4, F(t-4)+10, F(t-6)+17\}$$

With $F(0) = 0$ and $F(t) = -\infty$ for $t < 0$.

- Find the solution vector $X = (x_1, x_2, x_3, x_4, x_5)$, X(17) for MP(17), and X(14) for MP(14) **2 pts**

We use the algorithm:
$x = 0$ vector, $x = (0, 0, 0, 0, 0)$.
while $t > 0$ do;
if p(t)=k then { $x_k \leftarrow x_k + 1; t \leftarrow t - a_k$}
endwhile

X(15): t=15, p(15)=5, x=(0,0,0,0,0), $x_5 \leftarrow x_5 + 1, t \leftarrow t - a_5 = 15 - 6 = 9$, x=(0,0,0,0,1)
p(9)=1, $x_1 \leftarrow x_1 + 1, t \leftarrow t - a_1 = 9 - 3 = 6$, x=(1,0,0,0,1)
p(6)=5, $x_5 \leftarrow +1, t \leftarrow t - a_5 = 6 - 6 = 0$, x=(1,0,0,0,2)

Since t=0 , we have X(15) as $x = (1, 0, 0, 0, 2)$
Notice that $a_1 + 2a_5 = 15$ and $w_1 + 2w_5 = 41$.

4. **LCS - largest Common Subsequence**. Given the sequence X= (B, A, C, D, A, B, A) and Y = (A, B, C, A, D, B, A, A), we want to find largest subsequence common to both sequences X and Y. The tableau of c(i,j) is given as

	j	0	1	2	3	4	5	6	7	8
i		y_j	A	B	C	A	D	B	A	A
0	x_i	0	0	0	0	0	0	0	0	0
1	B	0	$0\uparrow$	$1\nwarrow$	$1\leftarrow$	$1\leftarrow$	$1\leftarrow$	$1\nwarrow$	$1\leftarrow$	
2	A	0	$1\nwarrow$	$1\uparrow$	$1\uparrow$	$2\nwarrow$	$2\leftarrow$	$2\leftarrow$	$2\nwarrow$	
3	C	0	$1\uparrow$	$1\uparrow$	$2\nwarrow$	$2\uparrow$	$2\uparrow$	$2\uparrow$	$2\uparrow$	
4	D	0	$1\uparrow$	$1\uparrow$	$2\uparrow$	$2\uparrow$	$3\nwarrow$	$3\leftarrow$	$3\leftarrow$	
5	A	0	$1\nwarrow$	$1\uparrow$	$2\uparrow$	$3\nwarrow$	$3\uparrow$	$3\uparrow$	$4\nwarrow$	
6	B	0								
7	A	0								

i) Fill the empty cells in row 6 (except column 8) **2 pts**

ii) Find a largest common subsequence of X^5 and Y^7 **2 pts**
(Note: X^5 is the subsequence of X containing first 5 elements.

Solution

i) Fill the empty cells **2 pts**

	j	0	1	2	3	4	5	6	7	8
i		y_j	A	B	C	A	D	B	A	A
0	x_i	0	0	0	0	0	0	0	0	0
1	B	0	$0\uparrow$	$1\nwarrow$	$1\leftarrow$	$1\leftarrow$	$1\leftarrow$	$1\nwarrow$	$1\leftarrow$	$1\leftarrow$
2	A	0	$1\nwarrow$	$1\uparrow$	$1\uparrow$	$2\nwarrow$	$2\leftarrow$	$2\leftarrow$	$2\nwarrow$	$2\nwarrow$
3	C	0	$1\uparrow$	$1\uparrow$	$2\nwarrow$	$2\uparrow$	$2\uparrow$	$2\uparrow$	$2\uparrow$	$2\uparrow$
4	D	0	$1\uparrow$	$1\uparrow$	$2\uparrow$	$2\uparrow$	$3\nwarrow$	$3\leftarrow$	$3\leftarrow$	$3\leftarrow$
5	A	0	$1\nwarrow$	$1\uparrow$	$2\uparrow$	$3\nwarrow$	$3\uparrow$	$3\uparrow$	$4\nwarrow$	$4\uparrow$
6	B	0	$1\uparrow$	$2\nwarrow$	$2\uparrow$	$3\uparrow$	$3\uparrow$	$4\nwarrow$	$4\uparrow$	$4\uparrow$
7	A	0	$1\nwarrow$	$2\uparrow$	$2\uparrow$	$3\nwarrow$	$3\uparrow$	$4\uparrow$	$5\nwarrow$	$5\nwarrow$

ii) Find a largest common sub-sequence of X^5 and Y^7 **2 pts**
(Note: X^5 is the sub-sequence of X containing first 5 elements.

Starting with cell c(5,7)=4 we have:
(5,7), (4,6), **(4,5)**, (3,4), **(2,4)**, (1,3), **(1,2)**, (0,1) reaching row 0. Thus we have, (in reverse order):
(5,7)=A, **(4,5)**=D, **(1,4)**=A, **(1,2)**=B. Reversing it we have **B A D A** .
Notice that X_1, X_2, X_4, X_5 and Y_2, Y_4, Y_5, Y_7 is B A D A .