Show your work! Give the best result that you can give!

1. Solve the following recursion via Master Theorem, if possible. Show your calculations and justify your results.

\[ T(n) = 4T(n/3) + \Theta(n^2) \]

\( \alpha = \log_3 4 = 1 < \alpha < 2, \beta = 2, h(n) = 1 \). We need to check regularity condition \( af(n/b) \leq cf(n) \) for some \( c < 1 \). Since we have \( 4(n/3)^2 = 4/9n^2 = cn^2 \) for \( c=4/9 < 1 \). Hence \( T(n) = n^2 = f(n) \)

\[ T(n) = 6T(n/2) + \Theta(n^2) \]

\( \alpha = \log_2 6 > 2 \) and \( \beta > 2 \). Hence \( T(n) = O(n^\alpha) \).

\[ T(n) = T(n/3) + \Theta(\log^3 n) \]

\( \alpha = \log_3 1 = 0 = \beta \). Since \( f(n) = h(n) = \log^k n \) for \( k=3 \) we have \( T(n) = O(f(n) \log n) = O(\log^4 n) \).

\[ T(n) = 2T(n/4) + \Theta(n \log n) \]

\( \alpha = \log_2 2 < 1 = \beta \). Since \( h(n) = \log n \), Master Theorem does not apply.

\[ T(n) = 3T(n/3) + \Theta(n/\log^2 n) \]

\( \alpha = \log_3 3 = 1 = \beta \). But, \( h(n) = \log^k n \) with \( k = -2 < 0 \). Thus Master Theorem does not apply.
2. **Order Statistics** Given a list of size 11 we want to find a median element of the list using partition algorithm. Take the first element of the list in question as the pivot element, and use the partition algorithm as discussed in class. For each recursive call to partition algorithm, indicate pivot element, (with a circle) the list input to partition algorithm, and order of the element which is sought (e.g. 5th, 7th etc). 

<table>
<thead>
<tr>
<th>n</th>
<th>i</th>
<th>list</th>
<th>partition</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>6</td>
<td>7 5 4 8 20 3 9 14 17 25 40</td>
<td>(3 5 4 ) 7 (20 8 9 14 17 25 40 )</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>20 8 9 14 17 25 40</td>
<td>(17 8 9 14 ) 20 (25 40 )</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>17 8 9 14</td>
<td>(14 8 9 ) 17 ()</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>14 8 9</td>
<td>(9 8 14 )</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>9 8</td>
<td>(8 ) 9 ()</td>
</tr>
</tbody>
</table>

Thus 9 is the median as (7 5 4 8 3 ) 9 (20 14 17 25 40 )

3. **Hashing- Double Hashing** Let \( h_1(k) = k \mod 16 \) and \( h'_2(k) = k \mod 11 \) and \( h_2(k) = h'_2(k) \) if \( h'_2(k) \) is odd, and \( h_2(k) = h'_2(k) + 1 \) otherwise. Let \( h(k,i) = (h_1(k) + ih_2(k)) \mod 16 \). Using the sequence \( h(k,i) \) \( i=0, 1, 2, \ldots \) place following in a Hash table: 5, 9, 21, 37, 29, 53, 4, 20, 36, 7.

<table>
<thead>
<tr>
<th>k</th>
<th>5</th>
<th>9</th>
<th>21</th>
<th>37</th>
<th>29</th>
<th>53</th>
<th>4</th>
<th>20</th>
<th>36</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_1(k) )</td>
<td>5</td>
<td>9</td>
<td>5</td>
<td>5</td>
<td>13</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>( h'_2(k) )</td>
<td>5</td>
<td>9</td>
<td>10</td>
<td>4</td>
<td>7</td>
<td>9</td>
<td>4</td>
<td>9</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>( h_2(k) )</td>
<td>5</td>
<td>9</td>
<td>1</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>5</td>
<td>9</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>( h(k,i)/i )</td>
<td>5/0</td>
<td>9/0</td>
<td>6/1</td>
<td>10/1</td>
<td>13/0</td>
<td>14/1</td>
<td>4/0</td>
<td>15/3</td>
<td>0/4</td>
<td>12/4</td>
</tr>
</tbody>
</table>

Note that \( h(k,i) = j/t \) means key k is placed in slot j, and it is achieved for i=t. In other words \( (k + t \ast h_2(k)) \mod 16 = j \). Let A be name of the array from 0 to 15. Thus \( h(k,i)/i \) means \( A(j) = k \) Now let see the calculations:

- \( k=5, h(k,0)=5 \) A(5)=5
- \( k=9, h(k,0)=9 \) A(9)=9
- \( k=21, h(k,0)=5, h(k,1)=6, A(6)=21 \)
- \( k=37, h(k,0)=5, h(k,1)=10, A(10)=37 \)
- \( k=29, h(k,0)=13, A(13)=29 \)
- \( k=53, h(k,0)=5, h(k,1)=14, A(14)=53 \)
- \( k=4, h(k,0)=4, A(4)=4 \)
- \( k=20, h(k,0)=4, h(k,1)=13, h(k,2)=6, h(k,3)=15, A(15)=20 \)
- \( k=36, h(k,0)=4, h(k,1)=7, A(7)=36 \)
- \( k=7, h(k,0)=7, h(k,1)=14, h(k,2)=5, h(k,3)=12, A(12)=7 \)

Thus the final hash table is:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>